



A large, abstract, swirling pattern in shades of blue and white occupies the upper two-thirds of the cover. The pattern is highly textured and organic, resembling turbulent fluid motion or a complex cloud formation. It has a dark blue background with lighter, more luminous, and textured areas where the swirls are more pronounced.

Turbulent Flows

Stephen B. Pope

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